The fractional nonlinear impurity: A Green function approach

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Introduction

We use a lattice Green function approach to study the stationary modes of a linear/nonlinear (Kerr) impurity embedded in a periodic one-dimensional lattice where we replace the standard discrete Laplacian by a fractional one. The energies and the mode profiles are computed in closed form, for different fractional exponents and different impurity strengths. The energies of the impurity mode lie outside the linear band whose bandwidth decreases steadily as the fractional exponent decreases. For any fractional exponent values, there is always a single bound state for the linear impurity while for the nonlinear (Kerr) case, up to two bound states are possible, for impurity strengths above certain threshold. The energy of the linear mode (or that of the upper energy nonlinear one), becomes directly proportional to the impurity strength at large impurity strengths. The transmission of plane waves is also computed in closed form for several fractional exponents, and various impurity strengths. We observe that fractionality tends to increase the overall transmission. The selftrapping transition for the nonlinear impurity shifts to lower nonlinearity values as the fractional exponent is decreased. In both cases, linear and nonlinear, we observe a form of trapping at zero impurity strength, which can be explained by the near-degeneracy of the spectrum in the limit of a small fractional exponent.

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References

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