

The fractional nonlinear electrical lattice

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Introduction

We examine the linear and nonlinear modes of a one-dimensional nonlinear electrical lattice, where the usual discrete Laplacian is replaced by a fractional discrete Laplacian. This induces a long-range intersite coupling that, at long distances, decreases as a power law. In the linear regime, we compute both, the spectrum of plane waves and the mean square displacement (MSD) of an initially localized excitation, in closed form in terms of regularized hypergeometric functions and the fractional exponent. The MSD shows ballistic behavior at long times, $\text{MSD} \sim t^2$ for all fractional exponents. When the fractional exponent is decreased from its standard integer value, the bandwidth decreases and the density of states shows a tendency towards degeneracy. In the limit of a vanishing exponent, the system becomes completely degenerate. For the nonlinear regime, we compute numerically the low-lying nonlinear modes, as a function of the fractional exponent. A modulational stability computation shows that, as the fractional exponent decreases, the number of electrical discrete solitons generated also decreases, eventually collapsing into a single soliton.

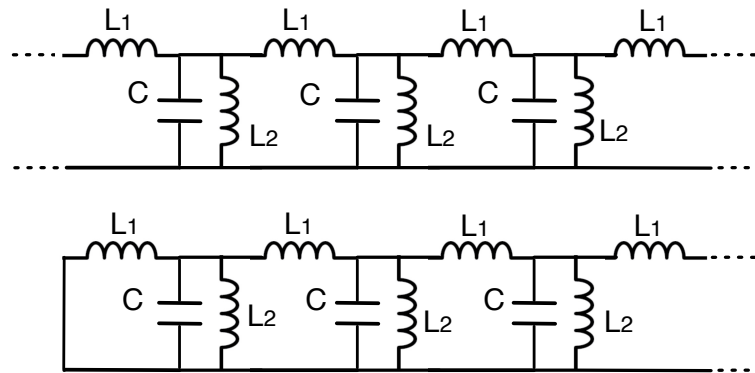


Figure 1: *Infinite (top) and semi-infinite (bottom) bi-inductive electrical lattice.*

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References

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