

# Redes complejas y la relación b-valor usando la distribución de probabilidad de conectividad: El caso de tres mega-terremotos en Chile en la última década.

Fernanda Martín<sup>1,2\*</sup>, Denisse Pastén<sup>1,2†</sup>

<sup>1</sup> Departamento de física, facultad de ciencias, Universidad de Chile.

<sup>2</sup> Grupo de Plasmas, complex networks and seismology (PLANETS).

\*fernanda.martin@ug.uchile.cl, †denisse.pasten.g@gmail.com

Studies from complex networks have increased in recent years, and different applications have been utilized in geophysics. Seismicity represents a complex and dynamic system that has open questions related to earthquake occurrence. In this work, we carry out an analysis to understand the physical interpretation of two metrics of complex systems: the slope of the probability distribution of connectivity ( $\gamma$ ) and the betweenness centrality (BC). To conduct this study, we use seismic datasets recorded from three large earthquakes that occurred in Chile: the  $M_w$ 8.2 Iquique earthquake (2014), the  $M_w$ 8.4 Illapel earthquake (2015) and the  $M_w$ 8.8 Cauquenes earthquake (2010). We find a linear relationship between the  $b$ -value and the value of  $\gamma$  with an interesting finding about the ratio between the  $b$ -value and  $\gamma$  that gives a value of  $\sim 0.4$ . We also explore a possible physical meaning of the BC. As a first result, we find that the behaviour of this metric is not the same for the three large earthquakes, and it seems that this metric is not related to the  $b$ -value and coupling of the zone. We present the first results about the physical meaning of metrics from complex networks in seismicity. These first results are promising, and we hope to be able to carry out further analysis to understand the physics that these complex network parameters represent in a seismic system.

The seismic data were measured by the National Seismological Center of Chile (Centro Sismológico Nacional, CSN)[1] between January 2005 and March 2017, containing 12 years of measurements with 38,083 seismic events in a zone between  $17.9^\circ$  and  $39.1^\circ$  South Latitude and between  $67.5^\circ$  and  $75^\circ$  West Longitude, with a maximum depth of 200.0 km. Therefore, we analyse the  $b$ -value, the critical exponent  $\gamma$  and the betweenness centrality for the three large earthquakes greater than  $M_w$ 8.0 that occurred in Chile in 2010, 2014 and 2015. From the initial large dataset measured between 2005 and 2017, we study the seismic events closer to the rupture zone of the main earthquakes, i.e., we consider the rectangle formed between the latitude coordinates that includes the longitude of the rupture zone for each earthquake, and in longitude coordinates, we use the complete seismic events that occurred in each zone. This decision is based on the amount of data collected in the original data; in this way, we have a large amount of data for the analysis.

We analyse three large earthquakes measured in Chile to understand if there is a relationship between the complex network theory and the physical processes that acted in the earthquake occurrences, the principal results are shown in table 1. We compute the values of the magnitude of completeness for each seismic zone studied:  $M_w$ 3.0 for the Cauquenes earthquake,  $M_w$ 2.9 for the Iquique earthquake and  $M_w$ 3.0 for the Illapel earthquake. The study on the  $b$ -value

gives different values for the three seismic datasets:  $b = 0.964$  for the Cauquenes earthquake,  $b = 0.758$  for the Iquique earthquake and  $b = 0.949$  for the Illapel earthquake. These results suggest a greater stress for the zone of the Iquique earthquake and a low value of stress for the Cauquenes and Illapel earthquakes. After this first seismic analysis, we carry out a complex network study finding a scale-free behaviour for the probability distribution of connectivity and for the cumulative distribution of the betweenness centrality. From complex networks, we find different values of the slope  $\gamma$  for the probability distribution of the degree. We use two methods to compute this slope, MLE and LR, in the case of the critical exponent  $\gamma$ , and the best results are from MLE. We obtain  $\gamma = 2.3$  for the Cauquenes earthquake,  $\gamma = 1.8$  for the Iquique earthquake and  $\gamma = 2.1$  for the Illapel earthquake.

**Table 1:** Table with the values of  $b$ , the Gutenberg-Richter law exponent, the value of  $\gamma$ , the characteristic exponent of probability distribution of connectivity, and the value of  $\delta$  exponent of the distribution of betweenness centrality of each earthquake. The exponents  $\delta$  and  $\gamma$  were calculated using the maximum likelihood method (MLE) and a simple linear regression (LR), and the value of  $b$  was calculated using only a linear regression (LR).

Zone	$M_c$	$b$	$\gamma$ LR	$\gamma$ MLE	$\delta$ LR	$\delta$ MLE
Cauquenes	3.0	$0.964 \pm 0.006$	$2.3 \pm 0.1$	$2.3 \pm 0.1$	$1.44 \pm 0.04$	$1.79 \pm 0.02$
Iquique	2.9	$0.758 \pm 0.004$	$1.9 \pm 0.1$	$1.9 \pm 0.1$	$1.89 \pm 0.05$	$1.76 \pm 0.03$
Illapel	3.0	$0.949 \pm 0.004$	$2.1 \pm 0.1$	$2.2 \pm 0.1$	$1.68 \pm 0.02$	$1.71 \pm 0.01$

If we calculate the ratio  $\frac{b}{\gamma}$  for each seismic zone, we find a particular number close to 0.4. These results suggest a linear relationship between the physical parameter  $b$  for the magnitude distribution of earthquakes and  $\gamma$ , a critical exponent from complex network theory. This is a work already published in the journal Entropy [2] and other results are shown within it.

**Agradecimientos:** The authors thank the National Seismological Center (Centro Sismológico Nacional).

## Referencias

- [1] CSN: Centro Sismológico Nacional, Universidad de Chile. Available online: URL <https://www.sismologia.cl>.
- [2] F. Martín and D. Pastén, *Complex Networks and the b-Value Relationship Using the Degree Probability Distribution: The Case of Three Mega-Earthquakes in Chile in the Last Decade*, Entropy, 24(3), 337 (2022)