Temperature fluctuations in finite systems: Application to the one-dimensional Ising chain

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Introduction

The theory of superstatistics[1], originally proposed for the study of complex nonequilibrium systems, has recently been extended to studies of small systems interacting with a finite environment[2, 3], because such systems display interestingly similar statistical behavior. In both situations there are several applicable definitions of inverse temperature, either intrinsic or dependent on the statistical ensemble. For instance, for the well-known Ising model with short-range interactions, it is possible to have in some cases temperature fluctuations. These fluctuations have been described using the traditional techniques from statistical mechanics, and also suitable extensions or generalizations of Boltzmann-Gibbs statistical mechanics have been advanced.

Recently, in a very concrete application of statistical mechanics of finite systems, Ilin *et al*[4] show how one can take a piece L from N spins in total and describe it as a system in contact with an environment that is commensurate with the system. Their results show how the system is described by a non-Gibbsian distribution that reduces to the traditional canonical ensemble in the limit when $L \ll N$, as expected.

Accordingly, in this work we propose to use the properties of the fundamental and the microcanonical inverse temperatures defined for nonequilibrium steady states to explore and study, as a particular case, the behavior of a one-dimensional Ising subsystem, being a part of an isolated Ising chain. Under the constraints that the superstatistical theory imposes on the two temperature functions, fundamental and microcanonical, we show that the non-Gibbsian distribution that describes the Ising subsystem is not consistent with superstatistics, despite having temperature fluctuations. In our results we highlight the importance of an environment with negative heat capacity as a necessary condition for superstatistics and the invariance of a new quantity, the inverse temperature covariance $\mathcal{U}$, between the full system, the subsystem and the environment that suggests the possibility of describing regions of an isolated system using new families of statistical models.

Acklowledgments:
The authors gratefully acknowledge funding from ANID FONDECYT Regular 1220651. CF also acknowledges Beca ANID Doctorado Nacional/(2021) - 21210658.

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