Logic and inference using Monte Carlo Metropolis simulation

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Introduction

When working with probabilities of logical propositions where not all the independent probabilities are known, it is not possible to obtain the remaining probabilities using traditional methods, and the use of techniques from statistical mechanics can bring additional insight. In this work we show how, using the maximum entropy principle [1], it is possible to assign probabilities from incomplete information. For instance, given the known probabilities of P(A|I) and P(A∧B|I), the entropy maximization leads to

$$P(a,b|I) = P(a,b|I_0)\cdot\frac{exp(-\lambda\cdot a-\mu\cdot a\cdot b)}{Z(\lambda,\mu)} \quad (1),$$

where Z is the partition function. As the number of states increases, the calculation of Z becomes a difficult task. From this problem arises the need for new accurate and efficient numerical methods. We avoid the computation of the partition function by means of Monte Carlo Metropolis simulation [2]. For this we propose new states using a random sample for a and b and the change is accepted with a given probability depending on λ and μ.

Applying this idea to both multipliers and iterating a sufficient amount of times the system converges to the original values of probabilities, and thus obtaining the Lagrange multipliers of the state. We obtain good agreements between the results and the theoretical model, as shown on Fig 1.

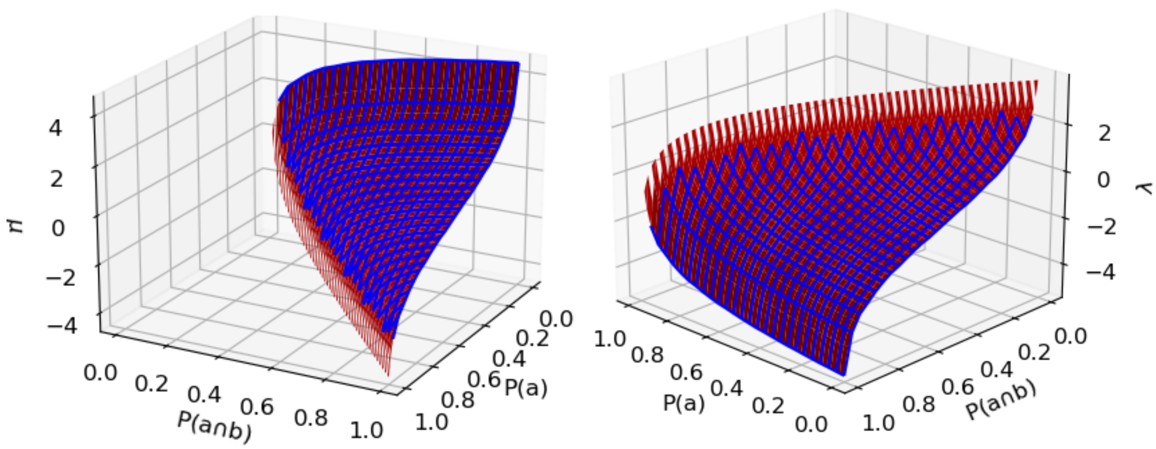

Figure 1: Theoretical values of multipliers (red surface) and their computed values using Monte Carlo Metropolis (blue wireframe) for multipliers μ (left) and λ (right).

References

[1] D. S. Sivia and J. Skilling, "Data Analysis: A Bayesian Tutorial". Oxford University Press (2006).
[2] D. P. Landau and K. Binder, "A Guide to Monte Carlo Simulations in Statistical Physics". Cambridge University Press (2021).