Study of the dynamical stability of a low dimensional system of coupled anharmonic oscillators from two distinct dynamical approaches

Castillo. M. S.[1*], L. Velázquez [1†], F. A. Calderón[1‡]
[1]Departamento de Física, Universidad Católica del Norte, Av. Angamos 0610, Antofagasta.
∗msc796@alumnos.ucn.cl, †lvelazquez@ucn.cl, ‡fcalderon@ucn.cl

Abstract

The instability of Hamiltonian dynamics can be developed using the linearisation of the dynamics of two nearby trajectories, the so called Tangent Dynamics (TD) [1]. Another framework for studying Hamiltonian chaos has been proposed by Pettini and collaborators [2]. This method uses the reformulation of Hamiltonian dynamics in the language of Riemannian geometry, so that, the stability of a geodesic flow depends on the curvature properties of configurational space equipped by means of the Jacobi-Levi-Civita (JLC) equation, and effectively quantifies chaos using Lyapunov exponents. We present here a numerical study of Hamiltonian chaos considering these above frameworks [3,4]. Our interest is focussed in systems with low dimensionality, in particular, a system of coupled anharmonic oscillators of the type β-FPU Hamiltonian. The integration of the system dynamics showed weak chaoticity characterised by small values for the Lyapunov exponents. This result was accompanied by a rather surprising behaviour: Lyapunov exponents extracted from the time-step noise using an extrapolation process seem to decrease in the low-energy range. Also a contrasting behavior is observed between the TD formalism and the JLC formalism. Although TD formalism predicts chaos in the system for the $N = 2$ and $N = 3$ particle case, the JCL formalism exhibits chaos several orders of magnitude smaller for the $N = 3$ particle case, and nonexistent for the $N = 2$ particle case. This behaviour seems to be caused by the inability of the JLC equation to detect parametic resonance, an incorrect characterization of the metric chosen for the system, as well as an incomplete characterization of the space under consideration.

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