Accurate analytical approximation for the regular Bessel function J_2(x)

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Introduction

The Bassel functions are very important in several areas of physics [1,2] and approxiamtions to $J_2(x)$ have been determined by the multipoint quasi-rational technique, MPQA [3,4]. The main idea is to use simultaneously the power series as well as the asymptotic expansion, and to built an analytic function as a bridge between both expansions by using rational functions combined with elementary functions as trigonometrical, hyperbolic, exponential and other simple functions.

Methods

In this case the approximated function was constructed as

$$\tilde{J}_2(x) = \frac{x\left[(p_0 + p_2\sqrt{1+\lambda^4 x^2} + p_4 x^2)\sin x + (p_1 + p_3\sqrt{1+\lambda^4} x^2)x\cos x\right]}{8(1+\lambda^4 x^2)^{\frac{3}{4}}(1+qx^2)} \tag{1}$$

The parameters q, λ and $p_i (i = 0; 4)$ were determined using the well known power series and asymptotic expansion of $J_2(x)$. Once this parameter were obtained, the absolute error of $\tilde{J}_0(x)$ compared with $J_0(x)$ were very small with about three digits exact. In this case the absolute error was considered, because of the zeros of $J_0(x)$. However, the zeros were also computed for the two functions $\tilde{J}_0(x)$ and $J_0(x)$, and in this case the relative errors were also calculated. The maximum relative error was the second zero and it was about 0,0005. The relative errors in most of the zeros were lower than 0,0001. It is interesting to point out that in the design of the approximate function $J_2(x)$ has been done, keeping in mind the preservation of symmetries, and because of that, the approximate function is good for positive and negative values of x.
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References

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